

Fitted GLMM:

$\begin{cases} 1 & \text{prog.} \\ 0 & \text{else} \end{cases}$  4/28/22

$$\log \left( \frac{\hat{E}(Y_{ij} | b_i)}{T_{ij}} \right) = 1.071 + 0.0512 X_{1ij}$$

4 fixed effect coeffs

$$- 0.0004996 X_{2ij} - 0.3062 X_{1ij} X_{2ij}$$

$\begin{cases} 1 & \text{post-besr} \\ 0 & \text{else} \end{cases}$

$$+ b_{0i} + b_{2i} X_{2ij}$$

Interpreting coeffs  $\rightarrow$  "Typical individual"  
( $b_{0i} = b_{2i} = 0$ )

- or when comparing two individuals  $\rightarrow$  for same rand. effects,

Variances (est.) of rand. effects:

$$\hat{\text{Var}}(b_{0i}) = 0.4999 \quad \leftrightarrow \quad \hat{\text{SD}}(b_{0i}) = 0.7070$$

$$\hat{\text{Var}}(b_{2i}) = 0.2319 \quad \leftrightarrow \quad \hat{\text{SD}}(b_{2i}) = 0.4816$$

$$\hat{\text{Cov}}(b_{0i}, b_{2i}) = 0.16 \quad \text{3 variance/covariance parameters}$$

= 4 + 3 = 7 total parameters

Individual subject w/ rand. effects  $\underline{b}_i = \begin{pmatrix} b_{0i} \\ b_{2i} \end{pmatrix}$

① Placebo at baseline:

$$\frac{E(Y_{ij} | \underline{b}_i)}{T_{ij}} = \exp(\beta_0 + b_{0i})$$

Seizure rate per week

② Progabide at baseline

$$\frac{E(Y_{ij} | b_{0i})}{T_{ij}} = \exp(\beta_0 + b_{0i} + \beta_1)$$

③ Placebo post-baseline

$$\frac{E(Y_{ij} | b_{0i})}{T_{ij}} = \exp(\beta_0 + b_{0i} + \beta_2 + b_{2i})$$

④ Progabide post-baseline

$$\frac{E(Y_{ij} | b_{0i})}{T_{ij}} = \exp(\beta_0 + b_{0i} + \beta_1 + \beta_2 + b_{2i} + \beta_3)$$

Interpreting  $\beta_2$ :

Seizure rate indiv. placebo post-baseline

Seizure rate indiv. placebo at baseline

$$= \frac{\exp(\beta_0 + b_{0i} + \beta_2 + b_{2i})}{\exp(\beta_0 + b_{0i})} = \exp(\beta_2 + b_{2i})$$

# Estimates of Sample Variance

$$s^2 = \frac{\sum (y_i - \bar{y})^2}{n-1}$$

- Restricted maximum likelihood
- ↳ random effects (variances)

$$S_{MLE}^2 = \frac{\sum (y_i - \bar{y})^2}{n}$$

- ↳ biased
- Maximum likelihood
- ↳ fixed effects ( $\beta$ 's)

Do we need  $b_{2i}$ ?

$$H_0: \sigma_2^2 = 0, \sigma_{02} = 0$$

Marginal Model -  $Y_{ij}$  =  $j^{\text{th}}$  observation on individual  $i$

- ↳ specify  $E(Y_{ij})$ ,  $\text{Var}(Y_{ij})$ ,  $\text{Cov}(Y_{ij}, Y_{ik})$   $j \neq k$
- no distributional assumptions

$$\log(E(Y_{ij})) = \beta_0 + \beta_1 X_{1ij} + \beta_2 X_{2ij} + \beta_3 X_{1ij} X_{2ij}$$

$$\underline{Y}_i = \begin{pmatrix} Y_{i1} \\ Y_{i2} \\ Y_{i3} \\ Y_{i4} \end{pmatrix}$$

$$\text{Cov}(\underline{Y}_i) = \begin{bmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{bmatrix}$$

$$\text{Var}(E(Y_{ij})) = E(Y_{ij})$$

Compound Symmetry

+  $\log(T_{ij})$