Travel Data Example

4/5/22

mode ~ hinc + psize

Baseline: Y = air

x_1 = household income (\$1000)

x_2 = party size

Fitted model:

$$\log\left(\boxed{\frac{\hat{\pi}_T}{\hat{\pi}_A}}\right) = \exp\left[1.550 - 0.0609\,x_1 + 0.2907\,x_2\right]$$

$$\log\left(\frac{\hat{\pi}_B}{\hat{\pi}_A}\right) = 1.034 - 0.0339\,x_1 - 0.3397\,x_2$$

$$\log\left(\frac{\hat{\pi}_C}{\hat{\pi}_A}\right) = -0.944 - 0.00354\,x_1 + 0.6006\,x_2$$

What about estimated conditional odds of train travel vs. car travel?

$$\log\left(\frac{\hat{\pi}_T/\hat{\pi}_A}{\hat{\pi}_C/\hat{\pi}_A}\right) = \log\left(\hat{\pi}_T/\hat{\pi}_A\right) - \log\left(\hat{\pi}_C/\hat{\pi}_A\right)$$

$$= (1.550 + 0.944) + (-0.0609 + 0.00354)\,x_1$$

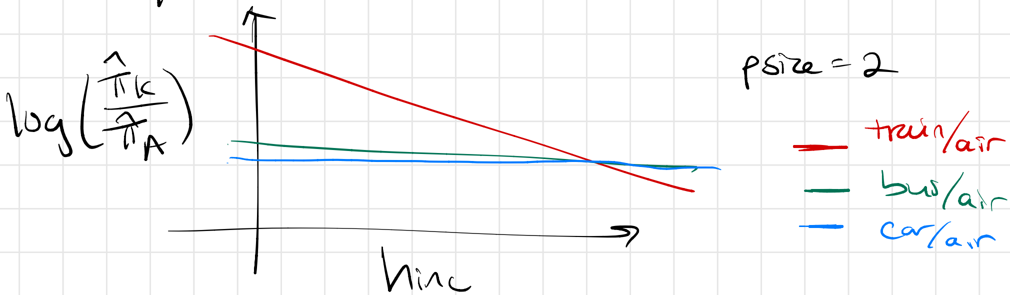
$$+ (0.2907 - 0.6006)\,x_2$$

$$= 2.49 - 0.058\,x_1 - 0.31\,x_2$$

Visualizing — plot of probabilities

← different for each psize

Plot log cond. odds



W/no interaction → intercepts change w/psize
but not slopes

Linear combinations of coefs.

95% CI for change in RRR of train to air for a \$1000 increase in hinc for parties of size 2

$$\frac{\pi_T}{\pi_A}$$

x_1 = household income (\$1000)

x_2 = party size

$$\log\left(\frac{\pi_T}{\pi_A}\right) = \beta_{01} + \beta_{11} x_1 + \beta_{21} x_2 + \beta_{31} x_1 x_2$$

$x_1 \uparrow 1$

$x_2 = 2$

$$\beta_{01} + \beta_{11} x_1 + \beta_{21}(2) + \beta_{31} x_1 (2)$$

$$= \beta_{01} + \beta_{21}(2) + \underline{(\beta_{11} + 2\beta_{31}) x_1}$$

95% CI for $e^{\beta_{11} + 2\beta_{31}}$: $(0.926, \ 0.973)$

-7.4% $\quad -2.7\%$

Interpret:

We are 95% confident that the true change in relative risk ratio of travel by train to air for a \$1000 increase in household income is between a 2.7% to 7.4% decrease, among parties of size two.

π_T / π_A

We are 95% confident that the relative risk ratio of travel by train to air / the conditional odds of travel by train compared to air decreases by between 2.7% to 7.4% for each \$1000 increase in household income when the party size is 2 individuals.

Section 6.2 — Cumulative Logit Models for ordinal response

$$Y = \begin{cases} 1 & \pi_1 \\ 2 & \pi_2 \\ 3 & \pi_3 \\ \vdots & \vdots \\ J & \pi_J \end{cases}$$

prob.

← labels reflect natural ordering

e.g. low, med, high
– Likert scale

$$\pi_k = P(Y = k)$$

A cumulative probability is the probability of falling at or below a particular category:

$$P(Y \leq k) = \pi_1 + \pi_2 + \dots + \pi_k$$

Notes: ① $P(Y \leq J) = 1$

② $P(Y \leq 1) \leq P(Y \leq 2) \leq \dots \leq P(Y \leq J)$ ← reflects ordering

For $k = 1, 2, \dots, J-1$

$$\text{logit}(P(Y \leq k)) = \beta_{0k} + \beta_{1k} x_1 + \dots + \beta_{mk} x_m$$

"cumulative logit" – $\log\left(\frac{P(Y \leq k)}{1 - P(Y \leq k)}\right)$

$$= \log\left(\frac{P(Y \leq k)}{P(Y > k)}\right) = \log\left(\frac{\pi_1 + \pi_2 + \dots + \pi_k}{\pi_{k+1} + \dots + \pi_J}\right)$$