# Cumulative Logit Models (cont)

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Simple case: One predictor $x$

$$\text{logit}(P(Y \le k)) = \log\left(\frac{P(Y\le k)}{1-P(Y\le k)}\right) = \alpha_k + \beta_k x$$

$$k = 1, 2, \ldots, J-1$$

<u>Proportional odds property</u>: $\Rightarrow$ $\beta_k = \beta$ for all $k$

Why called "proportional odds"?

Consider the log(OR) of $x+1$ to $x$:

$$\log\left(\frac{P(Y\le k \mid x+1) \,/\, P(Y > k \mid x+1)}{P(Y \le k \mid x) \,/\, P(Y > k \mid x)}\right)$$

$$= \alpha_k + \beta(x+1) - [\alpha_k + \beta x] = \beta$$

does not depend on $k$

Log-odds ratio comparing $x_2$ to $x_1$ $(x_2 > x_1)$

$\longrightarrow$ $\beta(x_2 - x_1)$ $\longrightarrow$ Same proportionality constant for all categories $k$

<u>Forms of model equation</u>:

① log-odds scale: $\log\left(\frac{P(Y\le k)}{1-P(Y\le k)}\right) = \alpha_k + \beta x$

② Odds scale: $\frac{P(Y\le k)}{1-P(Y\le k)} = \exp(\alpha_k + \beta x)$

③ Cumulative prob. scale: $P(Y\le k) = \frac{e^{\alpha_k+\beta x}}{1+e^{\alpha_k+\beta x}}$

Find $P(Y = k)$:

$$P(Y \le k) - P(Y \le k-1) = P(Y=k)$$

Test of independence between $X$ & $Y$?
($X$ categorical w/ $I$ levels)

$\chi^2$ test of independence (Ch. 2)
$df = (J-1)(I-1)$

Cumulative logit model: $\text{logit}(P(Y \le k)) = \alpha_k + \beta x$

$H_0: \beta = 0$ → More powerful test than $\chi^2$

$H_a: \beta \neq 0$

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$$P(Y \le k) = \frac{e^{\alpha_k + \beta x} / e^{\alpha_k + \beta x}}{1 + e^{\alpha_k + \beta x} / e^{\alpha_k+\beta x}} = \frac{1}{e^{-(\alpha_k + \beta x)} + 1}$$