Poisson Reg. one categ. predictor – Crab data

μ = mean # of satellites per female crab

$$\log \hat{\mu} = 1.4069 - 0.2146\, x_M - 0.6061\, x_{DM} - 0.6913\, x_D$$

$$x_c = \begin{cases} 1 & \text{color } c \\ 0 & \text{else} \end{cases}$$

exponentiated → 4.083 0.807 0.545 0.501

Int: The predicted average number of satellites for light medium (LM) female crabs is 4.083 males per crab.

x_{DM}:

	$\log(\hat{\mu})$
LM	1.4069
M	1.4069 − 0.2146
DM	1.4069 − 0.6061
D	1.4069 − 0.6913

$$\hat{E}(Y|LM) = e^{1.4069}$$

$$\hat{E}(Y|DM) = e^{1.4069 - 0.6061} = e^{1.4069} \cdot e^{-0.6061}$$

Interpret: The predicted average number of satellites for dark medium female crabs is 45% lower than that of light medium crabs.

Treat color as quantitative?

$$\log \hat{\mu} = 1.714 - 0.273\, x \qquad x = \text{color} = \begin{cases} 1 & LM \\ 2 & M \\ 3 & DM \\ 4 & D \end{cases}$$

What % change in $\hat{\mu}$ for DM compared to LM?

$$\frac{\hat{E}(Y|DM)}{\hat{E}(Y|LM)} = \frac{e^{1.714 - 0.273(3)}}{e^{1.714 - 0.273(1)}} = e^{-2(0.273)} \approx 0.579$$

Poisson Regression with Rates

Response = count over some time frame / 1 crab /

- If time frames (ref. unit) changes across observations ⟶ can't compare counts directly

Model rate parameter: $\lambda = \frac{\mu}{t}$ ← mean count ← unit time

Often clinical trials: t = "person-year"

Example: # of hip fractures at a nursing home

λ = average number of hip fractures per person-year

T = average # of residents at nursing home per year

Y = # of hip fractures at nursing home per year

$\mu = E(Y) = \lambda T$

Poisson Model: $\underset{\sim}{\mu} = \lambda T \iff \lambda = \frac{\mu}{T}$

want to model this w/ our linear predictor

$$\log(\lambda) = \log\left(\frac{\mu}{T}\right)$$

$$= \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k$$

$$\iff \log(\mu) = \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k + \log T$$

$$\iff \log(\lambda) = \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k$$

offset term